Angle Bisector Theorem

What is Angle Bisector Theorem?

An angle bisector is a straight line drawn from the vertex of a triangle to its opposite side in such a way, that it divides the angle into two equal or congruent angles. Now let us see, what the angle bisector theorem is.

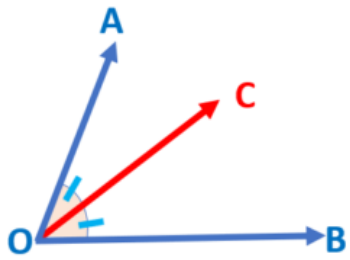
Angle Bisector

The concept of angle bisector is very important in geometry. Especially while learning geometry including triangles. In this section, we will learn angle bisector concepts and angle bisector theorem.

Angle Bisector

"An angle bisector is a ray or a line that divides an angle into two equal angles."

Let us look at the figure,



∠AOB is an angle and ray *OC*

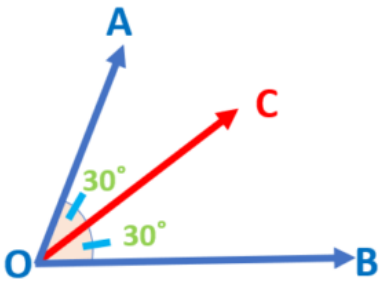
divides ∠AOB in such a way that

m∠AOC = m∠COB

and hence ray*OC* is the angle bisector of ∠AOB.

This is because ray*OC* divides ∠AOB into two equal angles.

For example,



Here,

∠AOB = 60º

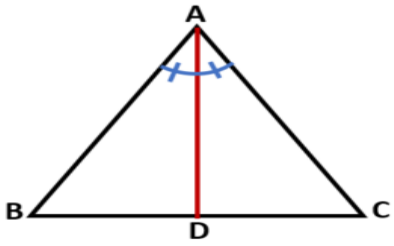
And ray *OC* divides it into two equal angles,

∠AOC = ∠COB = 30º

So, *OC* is the angle bisector.

Properties of an angle bisector

1. In a triangle, ***the angle bisector of any angle will divide the opposite side in the ratio of the sides containing the angle. This is also called ANGLE BISECTOR THEOREM of a triangle.***



In ΔABC,

AD is the angle bisector, such that ∠BAD = ∠CAD,

Then according to the angle bisector theorem,

Angle bisector of any angle will divide the opposite side in the ratio of the sides containing the angle.

Here, AD is the angle bisector, Sides AB and AC are containing the angle bisector.

BC is the opposite side and D divides it into two parts BD and DC,

So, according to the Angle bisector theorem.

$$\frac{BD}{DC} = \frac{AB}{AC}$$

Angle Bisector Theorem Proof

In a triangle, ***the angle bisector of any angle will divide the opposite side in the ratio of the sides containing the angle.***

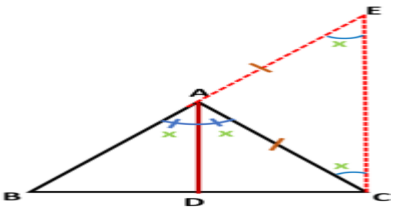
Given – In ΔABC, AD is the angle bisector, such that ∠BAD = ∠CAD .

To Prove –

$$\frac{BD}{DC} = \frac{AB}{AC}$$

Construction – Draw DA || CE to meet BA produced at E.

Proof –



Let,

∠BAD = ∠CAD = x

As DA || CE and line AC is the transversal, then

∠DAC = ∠ACE = x { Alternate interior angles } …….(i)

& ∠BAD = ∠BEC = x { Corresponding Angles } …….(ii)

Comparing equation (i) and (ii), we get

∠ACE = ∠AEC = x

Now, in ΔAEC,

As ∠ACE = ∠AEC = x

then AC = AE { Sides opposite to equal angles are equal } ……..(iii)

Now, in ΔBEC, As AD||EC

then by Thales theorem,

BD/ DC = BE/AE

DC/BD=AE/BE ……..(iv)

From equation (iii) AE = AC,

then putting this value in equation (iv)

BD/DC = AB/ AC

DC/BD=AC/AB

Hence proved.